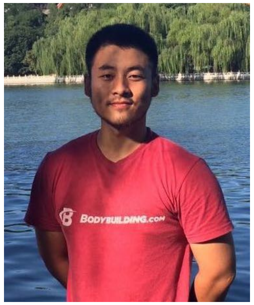
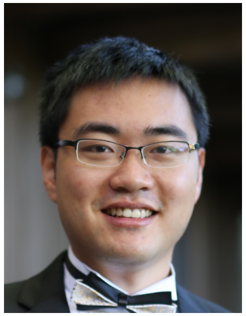
Theoretically Principled Trade-off between Robustness and Accuracy

Hongyang Zhang, TTIC

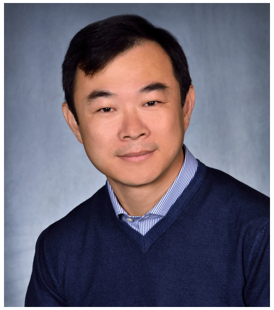
Yaodong Yu



Jiantao Jiao



Eric P. Xing



Laurent Ghaoui

Michael I. Jordan

Simons Institute for the Theory of Computing

Dec. 17th, 2019

Deep networks are unsafe

"Simons Institute"
87.7% confidence

$+ .007 \times$

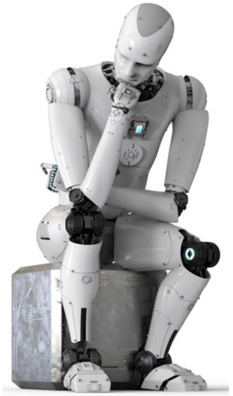
$=$

"Simons Institute"
99.3% confidence

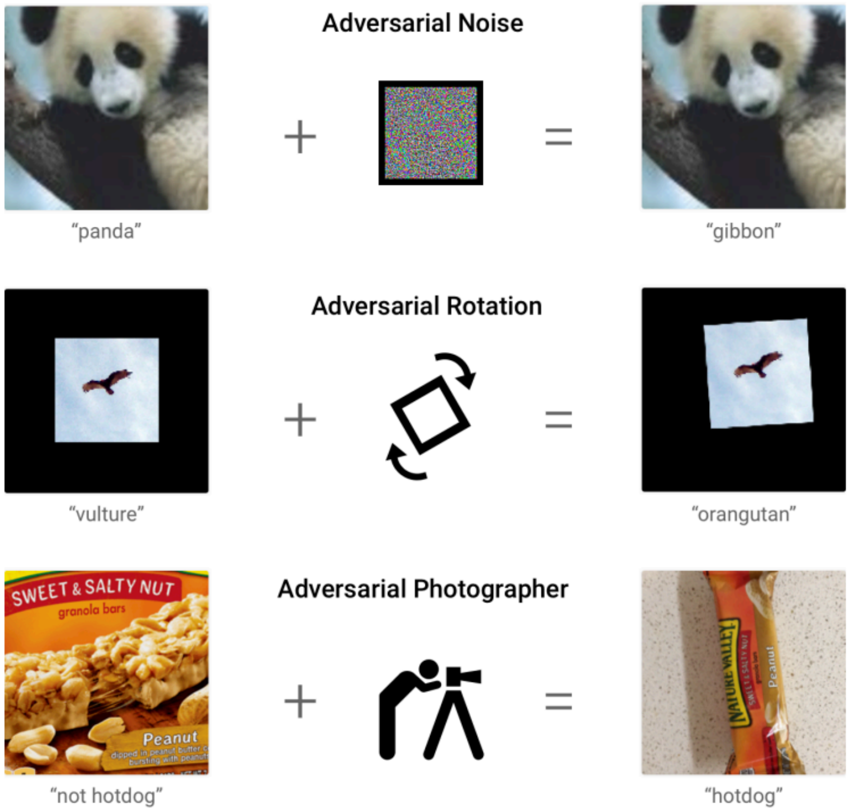
human
thinks

"TTIC"
99.3% confidence

machine
thinks



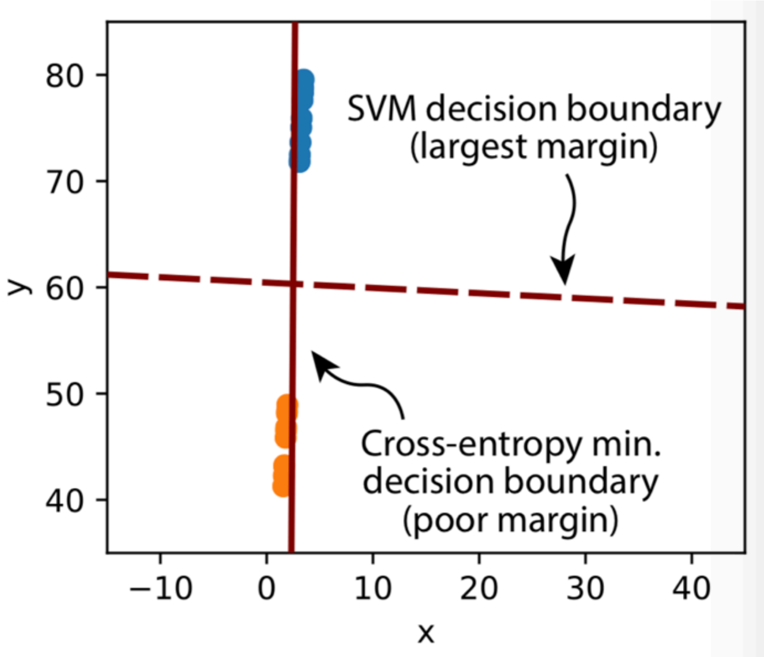
Deep networks are unsafe



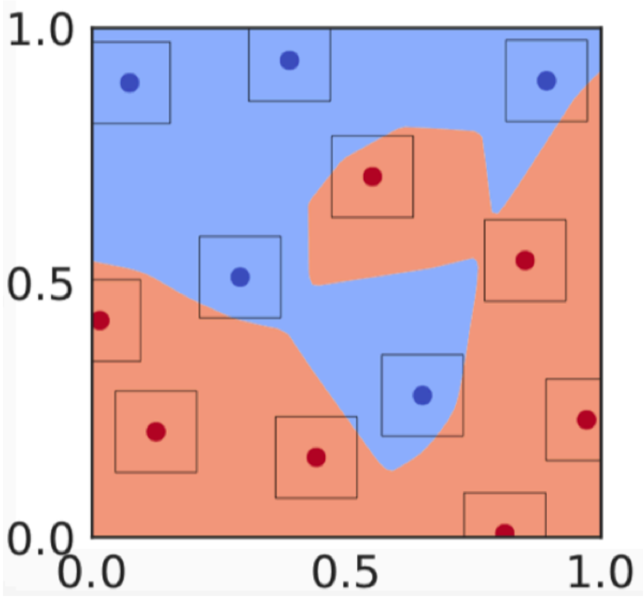
[BCZOCG'18] Unrestricted Adversarial Example, 2018

Why are there adversarial examples?

- We use a wrong loss function



Linear Case

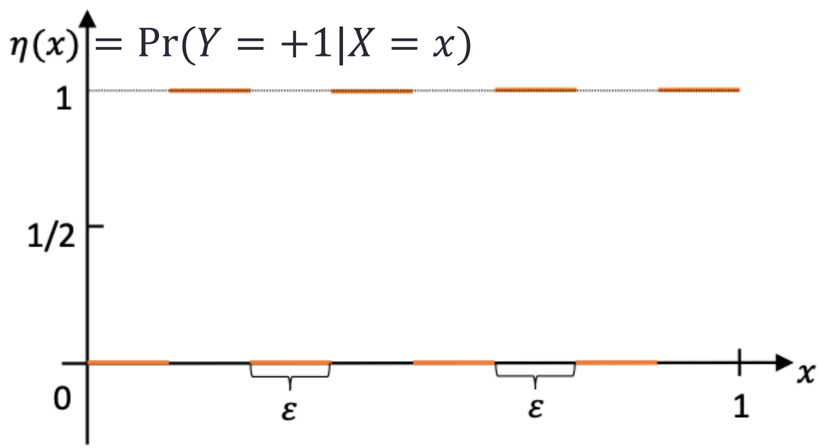


Non-Linear Case

Trade-off between Robustness and Accuracy

$$R_{rob}(f) := \mathbb{E}_{(X,Y)\sim D} 1\{\exists X' \in \mathbb{B}(X,\varepsilon)\ s.t.\ f(X')Y \le 0\}$$
$$R_{nat}(f) := \mathbb{E}_{(X,Y)\sim D} 1\{f(X)Y \le 0\}$$

- An example of trade-off:



$X \sim U[0,1]$

	Bayes Optimal Classifier	All-One Classifier
$\mathcal{R}_{\text{nat}}$	0 (optimal)	1/2
$\mathcal{R}_{\text{rob}}$	1	1/2 (optimal)

Solution: minimize **weighted average** $R_{nat}(f) + R_{rob}(f)/\lambda$!

Computationally, weighted average $R_{nat}(f) + R_{rob}(f)/\lambda$ is non-differentiable.

Surrogate Loss

- Classification-calibrated loss ϕ:

$$H(\eta) := \min_{\alpha \in \mathbb{R}} (\eta\phi(\alpha) + (1-\eta)\phi(-\alpha))$$

$$H^{-}(\eta) := \min_{\alpha : \alpha(2\eta-1) \leq 0} (\eta\phi(\alpha) + (1-\eta)\phi(-\alpha))$$

Definition (classification-calibrated loss):

ϕ is classification-calibrated loss, if for any $\eta \neq 1/2$, $H^{-}(\eta) > H(\eta)$.

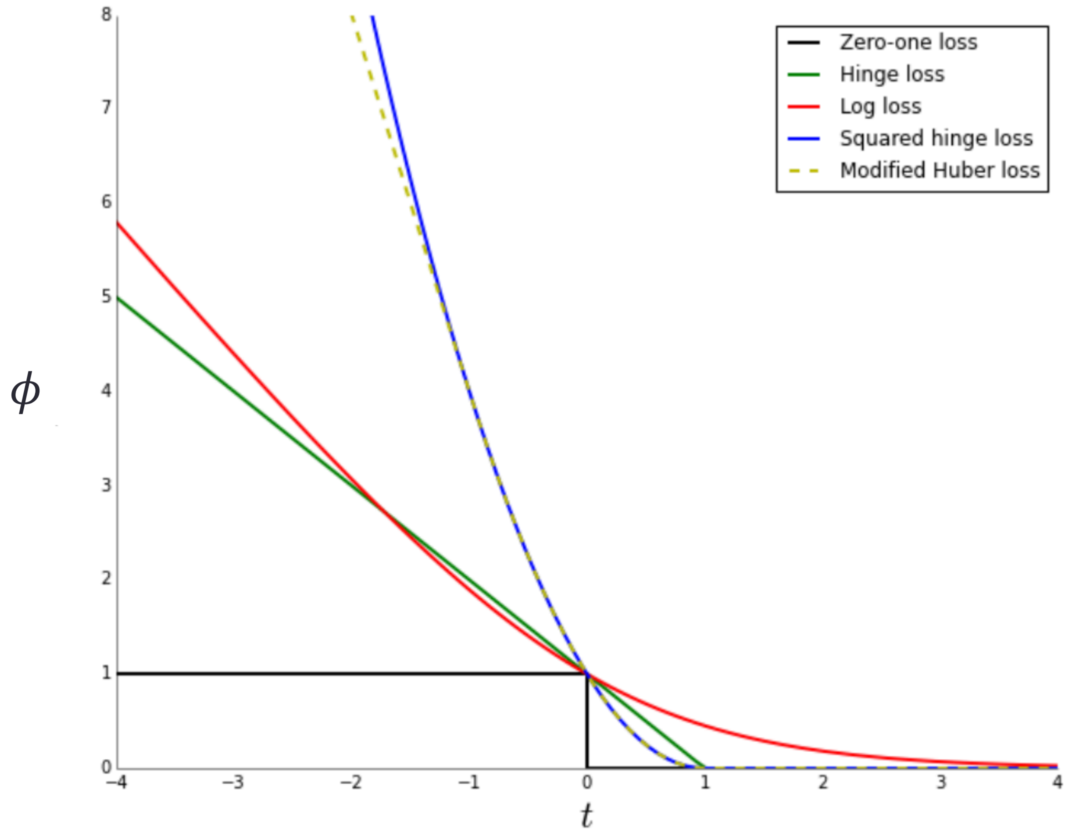
Intuitive explanation:

- Think about η as $\eta(x) = \Pr[Y = +1 | X = x]$, and α as score of positive class by f
- Then $H(\eta) = \min_f R_\phi(f)$

  $H^{-}(\eta) = \min_f R_\phi(f)$ s.t. f is inconsistent with Bayes optimal classifier
- Classification-calibrated loss: wrong classifier leads to larger loss for all $\eta(x)$

[BJM'06] Convexity, Classification, and Risk Bounds, 2006

Surrogate Loss



[BJM’06] Convexity, Classification, and Risk Bounds, 2006

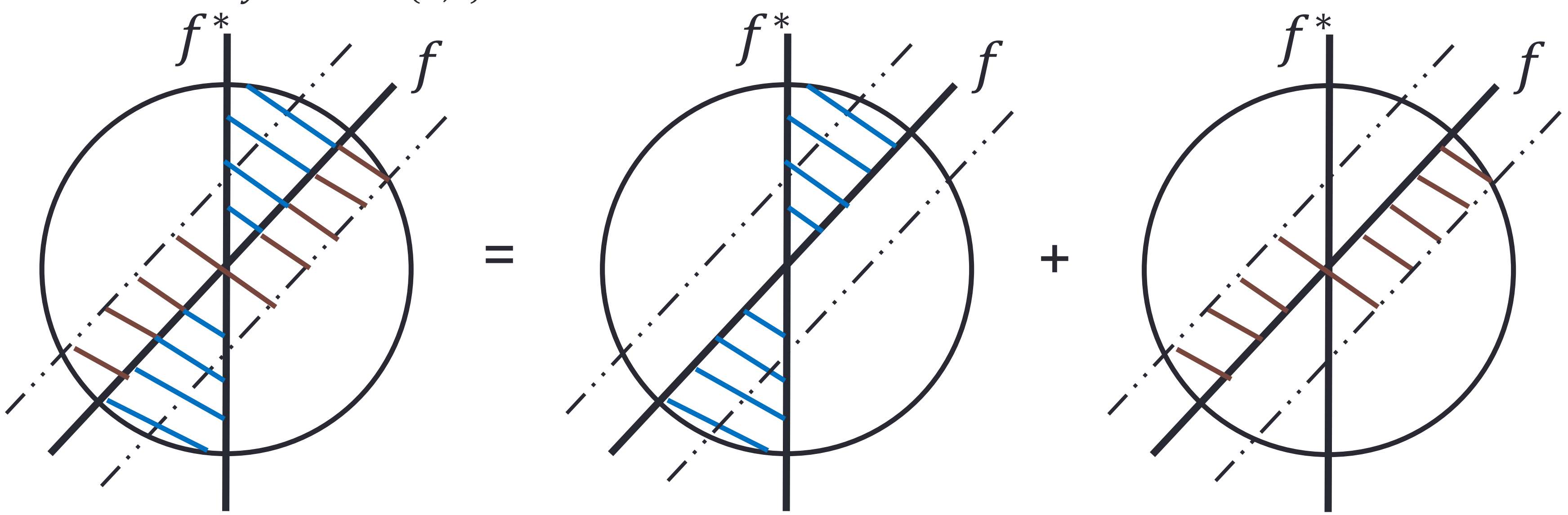
Main Results

Theorem 1 (Informal, upper bound, ZYJXGJ'19):

We have $R_{rob}(f) - R_{nat}^* \le R_\phi(f) - R_\phi^* + \mathbb{E} \max_{X' \in \mathbb{B}(X,\varepsilon)} \phi(f(X')f(X)/\lambda)$.

Proof Sketch:

- An important decomposition: $R_{rob}(f) = R_{nat}(f) + R_{bdy}(f)$
  where $R_{bdy}(f) = \mathbb{E}_{(X,Y)\sim D} 1\{\exists X \in \varepsilon \text{ neighbour of } f \text{ s.t. } f(X)Y > 0\}$



[ZYJXGJ'19] Theoretically Principled Trade-off between Robustness and Accuracy, ICML 2019

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  where $R_{bdy}(f) = \mathbb{E}_{(X,Y)\sim D} 1\{\exists X \in \varepsilon \text{ neighbour of } f \text{ s.t. } f(X)Y > 0\}$
- $R_{rob}(f) - R_{nat}^* = R_{nat}(f) - R_{nat}^* + R_{bdy}(f)$
- $R_{nat}(f) - R_{nat}^* \leq R_\phi(f) - R_\phi^*$ by [BJM'06]
- $R_{bdy}(f) \leq \mathbb{E} \max_{X' \in \mathbb{B}(X,\varepsilon)} 1(f(X')f(X) < 0) \leq \mathbb{E} \max_{X' \in \mathbb{B}(X,\varepsilon)} \phi(f(X')f(X)/\lambda)$

[BJM'06] Convexity, Classification, and Risk Bounds, 2006

[ZYJXGJ'19] Theoretically Principled Trade-off between Robustness and Accuracy, ICML 2019

Main Results

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Theorem 2 (Informal, lower bound, ZYJXGJ'19):

There exist a data distribution, a classifier f, and an $\lambda > 0$ such that

$$R_{rob}(f) - R_{nat}^* \geq R_\phi(f) - R_\phi^* + \mathbb{E} \max_{X' \in \mathbb{B}(X,\varepsilon)} \phi(f(X')f(X)/\lambda).$$

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Main Results

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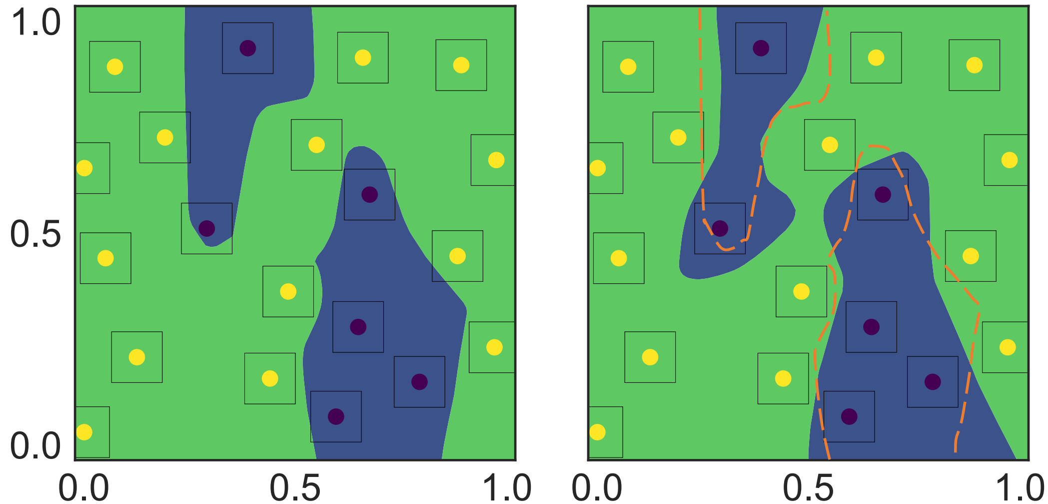
- New Surrogate Loss (TRADES):

$$\min_f [\mathbb{E}\, \phi(Yf(X)) + \mathbb{E} \max_{X' \in B_\varepsilon(X)} \phi(f(X)f(X')/\lambda)]$$



$\min_f \mathbb{E}\, \phi(Yf(X))$ $\quad$ $\min_f [\mathbb{E}\, \phi(Yf(X)) + \mathbb{E} \max_{X' \in B_\varepsilon(X)} \phi(f(X)f(X')/\lambda)]$

[ZYJXGJ'19] Theoretically Principled Trade-off between Robustness and Accuracy, ICML 2019

Significant Experimental Results

Experiments --- CIFAR10

Defense	Defense type	Under which attack	Dataset	Distance	$\mathcal{A}_{\text{nat}}(f)$	$\mathcal{A}_{\text{rob}}(f)$
[BRRG18]	gradient mask	[ACW18]	CIFAR10	0.031 (ℓ_∞)	-	0%
[MLW+18]	gradient mask	[ACW18]	CIFAR10	0.031 (ℓ_∞)	-	5%
[DAL+18]	gradient mask	[ACW18]	CIFAR10	0.031 (ℓ_∞)	-	0%
[SKN+18]	gradient mask	[ACW18]	CIFAR10	0.031 (ℓ_∞)	-	9%
[NKM17]	gradient mask	[ACW18]	CIFAR10	0.015 (ℓ_∞)	-	15%
[WSMK18]	robust opt.	FGSM^{20} (PGD)	CIFAR10	0.031 (ℓ_∞)	27.07%	23.54%
[MMS+18]	robust opt.	FGSM^{20} (PGD)	CIFAR10	0.031 (ℓ_∞)	87.30%	**47.04%**

$$\min_f \max_{X' \in B_\varepsilon(X)} \phi(Yf(X'))$$ (by Madry et al.)

TRADES ($1/\lambda = 1$)	regularization	FGSM^{20} (PGD)	CIFAR10	0.031 (ℓ_∞)	88.64%	49.14%
TRADES ($1/\lambda = 6$)	regularization	FGSM^{20} (PGD)	CIFAR10	0.031 (ℓ_∞)	84.92%	**56.61%**

$$\min_f [\mathbb{E}\,\phi(Yf(X)) + \mathbb{E} \max_{X' \in B_\varepsilon(X)} \phi(f(X)f(X'))/\lambda]$$ (ours)

TRADES ($1/\lambda = 6$)	regularization	LBFGSAttack	CIFAR10	0.031 (ℓ_∞)	84.92%	81.58%
TRADES ($1/\lambda = 1$)	regularization	MI-FGSM	CIFAR10	0.031 (ℓ_∞)	88.64%	51.26%
TRADES ($1/\lambda = 6$)	regularization	MI-FGSM	CIFAR10	0.031 (ℓ_∞)	84.92%	57.95%
TRADES ($1/\lambda = 1$)	regularization	C&W	CIFAR10	0.031 (ℓ_∞)	88.64%	84.03%
TRADES ($1/\lambda = 6$)	regularization	C&W	CIFAR10	0.031 (ℓ_∞)	84.92%	81.24%
[SKC18]	gradient mask	[ACW18]	MNIST	0.005 (ℓ_2)	-	55%
[MMS+18]	robust opt.	FGSM^{40} (PGD)	MNIST	0.3 (ℓ_∞)	99.36%	96.01%
TRADES ($1/\lambda = 6$)	regularization	FGSM^{40} (PGD)	MNIST	0.3 (ℓ_∞)	99.48%	96.07%
TRADES ($1/\lambda = 6$)	regularization	C&W	MNIST	0.005 (ℓ_2)	99.48%	99.46%

Interpretability

the class of bicycle

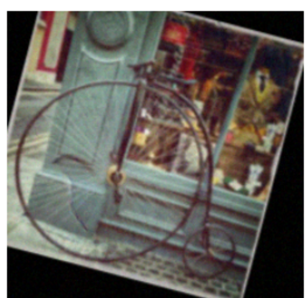
(a) clean example

(b) adversarial example by boundary attack with random spatial transformation

(c) clean example

(d) adversarial example by boundary attack with random spatial transformation

(e) clean example

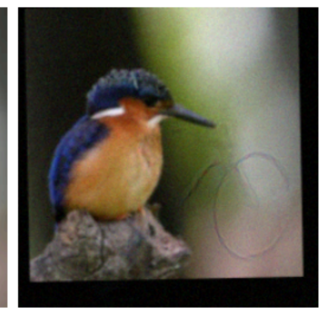


(f) adversarial example by boundary attack with random spatial transformation

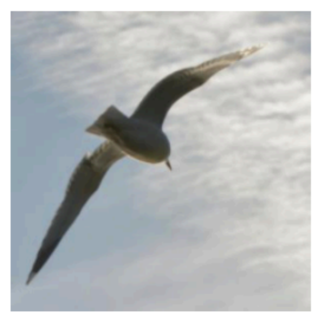
(a) clean example

(b) adversarial example by boundary attack with random spatial transformation

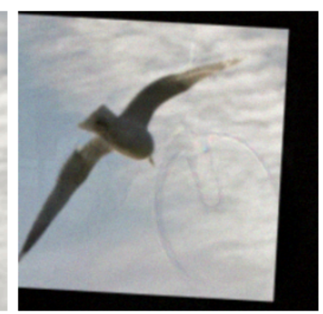
(c) clean example



(d) adversarial example by boundary attack with random spatial transformation



(e) clean example



(f) adversarial example by boundary attack with random spatial transformation

the class of bird

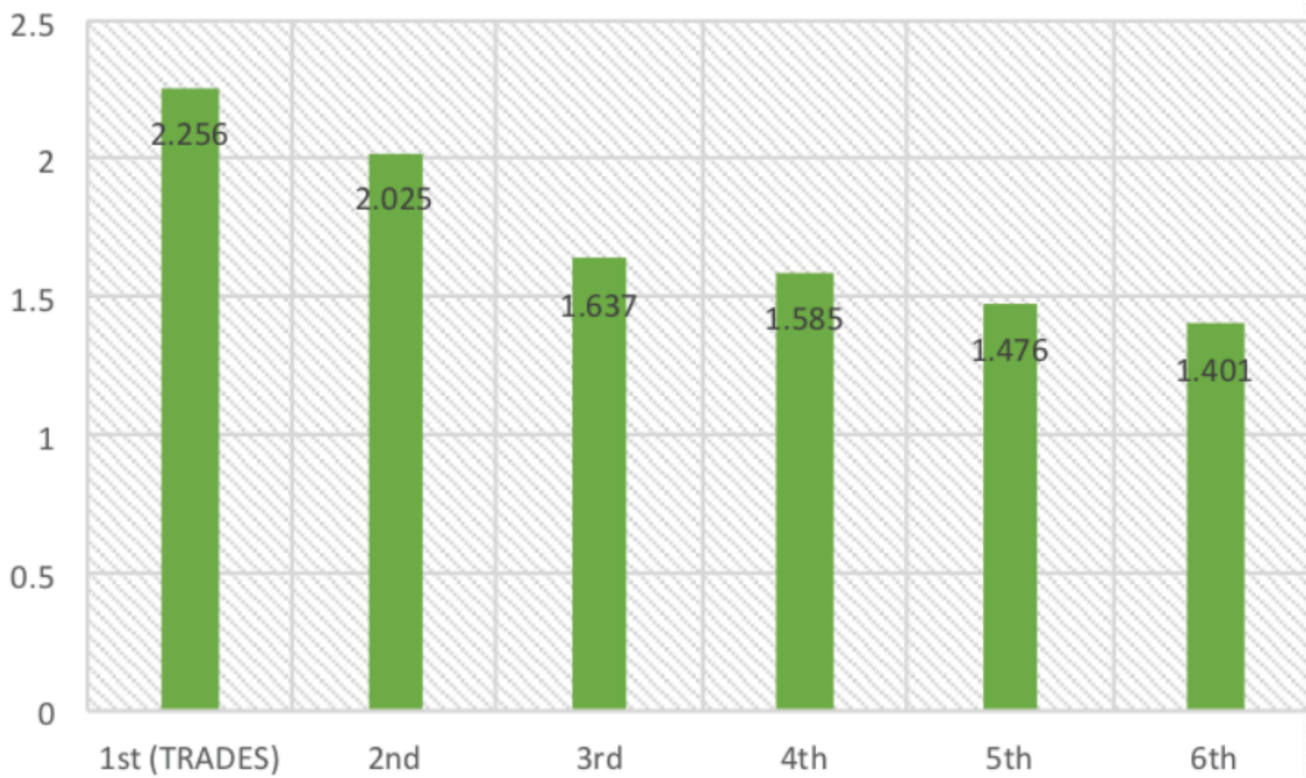
Competition: NeurIPS 2018 Adversarial Vision Challenge

- Evaluation criterion
  - 400+ teams, ~2,000 submissions
  - Tiny ImageNet dataset
  - Model Track and Attack Track
  - Participants in the two tracks play against each other

Competition: NeurIPS 2018 Adversarial Vision Challenge

Adversarial Vision Challenge
Model Track

- Final Result



Recent Developments of TRADES

- Acceleration [SNG+19,ZZL+19]:
  - Achieve 30x speed-up, almost as fast as natural training
- Semi-supervised learning/unlabel data [CRS+19,SFK+19]:
  - TRADES + self-training (500K) improves robustness by +5% on CIFAR10
- Applications [JHC+19]:
  - 1st place in Glue leaderboard (up until Dec. 9th) in NLP --- SMART
- Theoretical understanding (upcoming):
  - Benefits of local Lipschitzness
  - Provable certification of TRADES by random smoothing

[SNG+19] Adversarial training for free, 2019.
[ZZL+19] You only propagate once, 2019.
[CRS+19] Unlabel data improves adversarial robustness, 2019.
[SFK+19] Are labels requires for improving adversarial robustness?, 2019.
[JHC+19] SMART, 2019.

Conclusions

- Adversarial Robustness
  - Trade-off matters in the adversarial defense
  - Matching upper and lower bounds on $R_{rob}(f) - R^*_{nat}$
  - New surrogate loss for adversarial defense
  - Winners of NeurIPS 2018 Adversarial Vision Challenge
  - Some recent developments

Thank You

Trade-off between Robustness and Accuracy

- Our goal: Find a classifier $\hat{f}$ such that $R_{rob}(\hat{f}) \leq \mathrm{OPT} + \delta$

$$\mathrm{OPT} := \min_{f} R_{rob}(f), \qquad \text{s.t.} \qquad R_{nat}(f) \leq R_{nat}^{*} + \delta$$

$$\Downarrow$$

suffice to show $\boxed{R_{rob}(f) - R_{nat}^{*}} \leq \delta$

PyTorch Package

- New Surrogate Loss:

$$\min_f [\mathbb{E}\, \phi\big(Yf(X)\big) + \mathbb{E} \max_{X' \in B_\varepsilon(X)} \phi(f(X)f(X')/\lambda)]$$

Natural training:

```
def train(args, model, device, train_loader, optimizer, epoch):
    model.train()
    for batch_idx, (data, target) in enumerate(train_loader):
        data, target = data.to(device), target.to(device)
        optimizer.zero_grad()
        loss = F.cross_entropy(model(data), target)
        loss.backward()
        optimizer.step()
```

replace

Adversarial training by TRADES:

To apply TRADES, cd into the directory, put 'trades.py' to the directory.

```
from trades import trades_loss

def train(args, model, device, train_loader, optimizer, epoch):
    model.train()
    for batch_idx, (data, target) in enumerate(train_loader):
        data, target = data.to(device), target.to(device)
        optimizer.zero_grad()
        # calculate robust loss - TRADES loss
        loss = trades_loss(model=model,
                           x_natural=data,
                           y=target,
                           optimizer=optimizer,
                           step_size=args.step_size,
                           epsilon=args.epsilon,
                           perturb_steps=args.num_steps,
                           batch_size=args.batch_size,
                           beta=args.beta,
                           distance='l_inf')
        loss.backward()
        optimizer.step()
```

- Link: https://github.com/yaodongyu/TRADES

Competition II: Unrestricted Adversarial Example

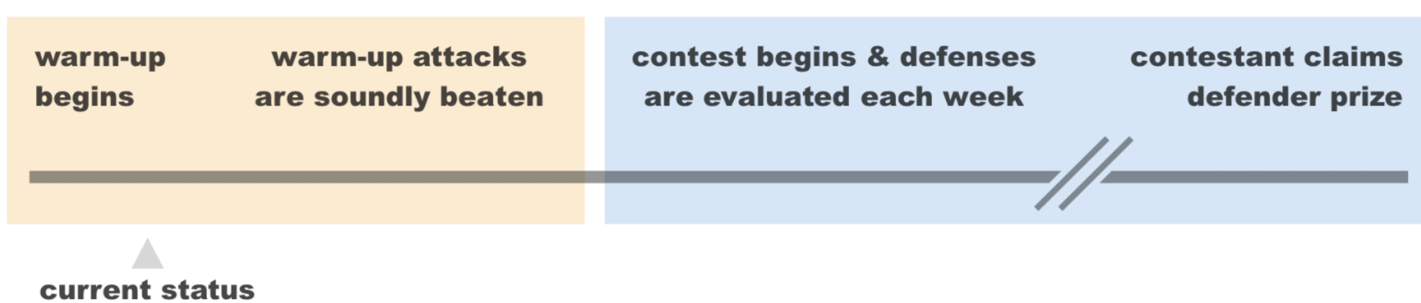
Unrestricted Adversarial Examples Challenge

build passing

In the Unrestricted Adversarial Examples Challenge, attackers submit arbitrary adversarial inputs, and defenders are expected to assign low confidence to difficult inputs while retaining high confidence and accuracy on a clean, unambiguous test set. You can learn more about the motivation and structure of the contest in our recent paper

This repository contains code for the warm-up to the challenge, as well as the public proposal for the contest. We are currently accepting defenses for the warm-up.

Warm-up & Contest Timeline



Competition II: Unrestricted Adversarial Example

Defense	Submitted by	Clean data	Common corruptions	Spatial grid attack	SPSA attack	Boundary attack	Submission Date
Pytorch ResNet50 (trained on bird-or-bicycle extras)	TRADESv2	100.0%	100.0%	99.5%	100.0%	95.0%	Jan 17th, 2019 (EST)
Keras ResNet (trained on ImageNet)	Google Brain	100.0%	99.2%	92.2%	1.6%	4.0%	Sept 29th, 2018
Pytorch ResNet (trained on bird-or-bicycle extras)	Google Brain	98.8%	74.6%	49.5%	2.5%	8.0%	Oct 1st, 2018

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Future Directions about Robustness

- Computational and Statistical Theory
  - Understand the optimization principal of new surrogate loss
  - (Tight) sample complexity of adversarial learning
- Applications of AI Security
  - Robotics, autonomous cars
  - Medical diagnose
- Extensions with other frameworks
  - Self-supervised/semi-supervised learning
  - Neural ODE